

## Model of the Physical Space from Quantum Mechanics

Otto C.W. Kong

*Department of Physics and Center for Mathematics and Theoretical Physics,  
National Central University, Chung-li, TAIWAN 32054  
E-mail: otto@phy.ncu.edu.tw*

**Abstract :** The physical world is quantum. However, our description of the quantum physics still relies much on concepts in classical physics and in some cases with ‘quantized’ interpretations. The most important case example is that of spacetime. We examine the picture of the physical space as described by simple, so-called non-relativistic, quantum mechanics instead of assuming the Newtonian model. The key perspective is that of (relativity) symmetry representation, and the idea that the physical space is to be identified as the configuration space for a free particle. Parallel to the case of the phase space, we have a model of the quantum physical space which reduces to the Newtonian classical model under the classical limit. The latter is to be obtained as a contraction limit of the relativity symmetry.

---

\* Talk presented at the Eighth International Workshop DICE 2016 — Spacetime, Matter, Quantum Mechanics (Sep 12 - 16), Castiglioncello, Italy.

# Model of the Physical Space from Quantum Mechanics

**Otto C W Kong**

Department of Physics and Center for Mathematics and Theoretical Physics, National Central University, Chung-li, Taiwan 32054

E-mail: [otto@phy.ncu.edu.tw](mailto:otto@phy.ncu.edu.tw)

**Abstract.** The physical world is quantum. However, our description of the quantum physics still relies much on concepts in classical physics and in some cases with ‘quantized’ interpretations. The most important case example is that of spacetime. We examine the picture of the physical space as described by simple, so-called non-relativistic, quantum mechanics instead of assuming the Newtonian model. The key perspective is that of (relativity) symmetry representation, and the idea that the physical space is to be identified as the configuration space for a free particle. Parallel to the case of the phase space, we have a model of the quantum physical space which reduces to the Newtonian classical model under the classical limit. The latter is to be obtained as a contraction limit of the relativity symmetry.

## 1. Conceptual Introduction

Newtonian mechanics is our first comprehensive model of a fundamental particle dynamics. Most of our theoretical conceptual notions in physics are tied to its formulation. As such, they are classical concepts. The physical world is quantum. Its description should be based on quantum concepts each of which may be very different from its classical counterpart, if there is at all a counterpart. Most if not all of our fundamental conceptual notions in physics as an intuitive, common sense, origin. However, such intuitive concepts do not fix their possible mathematical formulation which is an abstraction necessary for the analysis in a physical theory with precision. The formulation of such intuitive concepts into the concepts of classical physics is what physicists are familiar with, but not at all *a priori* more intuitive than their quantum counterparts. The unfortunate common miss-statement of quantum physics being less than intuitive has however hindered our appreciation of quantum physics with the more appropriate quantum formulation of the fundamental conceptual notions. Our model of space and time is the most central problem at hand. A common theme in the classical concepts is that the corresponding physical quantities, such as the position of a particle in space, have real number values. Quantum physics indicates otherwise, at least not necessarily by a finite number of real numbers. More generally, one should consider such quantities as having values modeled by some appropriate types of abstract algebraic entities. The system of real numbers as one of the latter has no preferred role apart from being our first order approximation to the physical nature as in the classical theory. We can summarize it all by saying that the main culprit in the limitation of the classical concept is the real number, which the philosopher W. Quine once called a ‘convenient fiction’; with quantum physics, the old fiction is not so convenient any more.

Let us focus on the notion of the physical space as a collection of all possible positions, either of a free particle or equivalently as the center of mass for a system of particles. The first mathematical model is given by the three dimensional Euclidean geometry, which had been the only available model for centuries till beyond Newton's time. And it was of course adopted by Newton in his mechanics. Again, any mathematical model is beyond naive intuition. When, as it stand today, we have more than one such model, which one is the correct, or simply better, model for the physical space is a physics question. The answer is to be seek from matching the corresponding theories to experimental result. The limited success of Newtonian mechanics says that the Newtonian model may be not good enough. We are now familiar with the Einstein models, for his theories of special and general relativity. However, the Einstein models are still limited to real number geometry. Like the Newtonian model, they are classical (physical) models. We want to look at quantum models. A simple natural guess based on modern mathematics would be a kind of topological space which likely has some noncommutative geometry [1]. The idea of quantum geometry is not new in the subject matter related to deep microscopic structure of spacetime and quantum gravity. However, we present here a model of the physical space as behind simple quantum mechanics and illustrate its having the correct classical limit [2, 3].

At this point, some readers may feel uncomfortable about our suggestion of going beyond the notion of a three dimensional manifold calling it nothing counter-intuitive. The following comment is in order. The notion of a three 'dimensional' space, or four 'dimensional' spacetime, is no doubt quite intuitive. The Chinese term for the universe as a totality of all spacetime, for example, has two characters the first of which is the space part which literally means extending along the three principle axes. However, it is more tri-axial than three dimensional. Nothing in our intuition points towards identifying a definite position along any axis, or equivalently a distance with the abstract mathematical entity called a real number. One can see that the notion of being sort of tri-axial stays in our model.

## 2. The Role of Relativity Symmetries

The 'particle' is the key concept for an ideal physical object in the Newtonian theory, one that still play a key role in simple quantum mechanics as a quantized version of the latter. To analyze the logic behind the Newtonian formulation, the first question is 'why particle?'. The particle is matter (with a single characteristic called mass) which has an unambiguous position in the Euclidean space. The Newton's laws are really definitions of notions like states (of motion), valid frames of reference, force, and mass (as inertia). Note that time is also modeled on an (one dimensional) Euclidean geometry. All the notions put together gives a model of physical phenomena the success of which obviously breaks down at the atomic scale and beyond. We argue that the modified theory of quantum mechanics has all the notions modified including those of position in space, though time is not touched so long as the 'non-relativistic' theory is concerned. We stick to the setting here.

In Ref.[2], we presented the perspective of taking the relativity symmetry as the starting point to look at the Newtonian model as well as formulating the quantum model and its classical limit. It is a perspective that is behind a big program aiming at eventually constructing the quantum spacetime model and its dynamics at the deep microscopic level [4, 5]. Symmetry is the single most important theoretical structure in modern physics. A relativity symmetry is the key symmetry behind a theory of mechanics. It is the group of reference frame transformation, and the symmetry of the spacetime model in the theory. The configuration space and phase space of a free particle are all natural homogeneous spaces as representations of the symmetry. The algebra of observables also carries a representation of it.

For the Newtonian theory, the relativity symmetry is given by the Galilei group. Newtonian space-time, as well as the single particle configuration and phase spaces can all be identified as coset spaces of the Lie group. The coset for the space-time is given by the quotient of the Galilei

group  $G(3)$  factored by the  $ISO(3)$  subgroup of rotations and boosts. The corresponding case for the Einstein theory is the Minkowski spacetime  $M^4 = ISO(1,3)/SO(1,3)$ , the coset space of the Poincaré group factored by the Lorentz group. In fact, the Newtonian limit is exactly to be recovered by a contraction of  $ISO(1,3)$  to  $G(3)$  as  $c \rightarrow \infty$  [6, 7]. What we have, as shown below, is much of a parallel picture for the quantum versus classical case. The action of  $G(3)$  on the coset is given by the familiar expressions

$$\begin{pmatrix} t' \\ x^i \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & B \\ V^i & R_j^i & A^i \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} t \\ x^j \\ 1 \end{pmatrix} = \begin{pmatrix} t+B \\ V^i t + R_j^i x^j + A^i \\ 1 \end{pmatrix}, \quad (1)$$

and the more convenient infinitesimal form

$$\begin{pmatrix} dt \\ dx^i \\ 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & b \\ v^i & \omega_j^i & a^i \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} t \\ x^j \\ 1 \end{pmatrix} = \begin{pmatrix} b \\ v^i t + \omega_j^i x^j + a^i \\ 0 \end{pmatrix}. \quad (2)$$

Kinematical structure alone is enough to look at the configuration and phase spaces on which generic dynamics can be described with a time translation generated by a Hamiltonian with a nontrivial potential. Hence, we first restrict our analysis the subgroup,  $G(3)$  with the time translations  $T$  taken out; denoted by  $G(3)_s$ . The Newtonian space can be identified with  $G(3)_s/ISO(3)$  which is the same as  $G(3)/ISO(3) \times T$ . For convenience in matching with the familiar quantum picture, we use  $X_i$  generators in place of the boost generators  $K_i = mX_i$ . The Newtonian space then has  $dx^i = \omega_j^i x^j + \bar{x}^i$ , and for the phase space  $(p^i, x^i)$ , we have also  $dp^i = \omega_j^i p^j + \bar{p}^i$ . The phase space is given by  $G(3)_s/SO(3) = G(3)/SO(3) \times T$ . The position and momentum coordinates  $x^i$  and  $p^i$  both transform as a three vector under rotations and have their own independent translations generated by  $P_i$  and  $X_i$ , respectively.

### 3. The Quantum Phase Space Is The Physical Space

The quantum symmetry differs from the classical one. We have the Heisenberg commutation relation which corresponds to having the Lie algebra to the Galilean symmetry modified by

$$[X_i, P_j] = i\delta_{ij}I, \quad (3)$$

where  $I$  is an extra central generator that commutes with all others. It is a bigger group  $\tilde{G}(3)$  which is a  $U(1)$  central extension of  $G(3)$  [8]. Without the time translations, it is a Heisenberg-Weyl group supplemented with the rotations denoted by  $H_R(3)$ . In the setting, one has naively the space coset  $H_R(3)/ISO(3)$  with

$$\begin{pmatrix} dx^i \\ d\theta \\ 0 \end{pmatrix} = \begin{pmatrix} \omega_j^i & 0 & \bar{x}^i \\ \bar{p}_j & 0 & \bar{\theta} \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x^j \\ \theta \\ 1 \end{pmatrix} = \begin{pmatrix} \omega_j^i x^j + \bar{x}^i \\ \bar{p}_j x^j + \bar{\theta} \\ 0 \end{pmatrix}, \quad (4)$$

and the phase space coset  $H_R(3)/SO(3)$  with

$$\begin{pmatrix} dp^i \\ dx^i \\ d\theta \\ 0 \end{pmatrix} = \begin{pmatrix} \omega_j^i & 0 & 0 & \bar{p}^i \\ 0 & \omega_j^i & 0 & \bar{x}^i \\ -\frac{1}{2}\bar{x}_j & \frac{1}{2}\bar{p}_j & 0 & \bar{\theta} \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} p^j \\ x^j \\ \theta \\ 1 \end{pmatrix} = \begin{pmatrix} \omega_j^i p^j + \bar{p}^i \\ \omega_j^i x^j + \bar{x}^i \\ \frac{1}{2}(\bar{p}_j x^j - \bar{x}_j p^j) + \bar{\theta} \\ 0 \end{pmatrix}. \quad (5)$$

Note that  $H_R(3)$  is the central extension of  $G(3)_s$  above. The funny cosets with a  $\theta$  coordinate apparently do not serve our purpose. However, we know how to construct the quantum phase

space from the second coset. That is the formulation of the Hilbert space of canonical coherent states [9].

The phase space coset is isomorphic to the Heisenberg-Weyl subgroup  $H(3)$ . A unitary representation of the latter is given by taking the linear span of the canonical coherent states each of which can be identified as a point in the coset. They are given by

$$e^{i\theta} |p^i, x^i\rangle = U(p^i, x^i, \theta) |0\rangle \quad (6)$$

where

$$U(p^i, x^i, \theta) \equiv e^{i\frac{x_i p^i}{2}} e^{i\theta \hat{I}} e^{-ix^i \hat{P}_i} e^{ip^i \hat{X}_i} = e^{i(p^i \hat{X}_i - x^i \hat{P}_i + \theta \hat{I})}, \quad (7)$$

and  $|0\rangle \equiv |0, 0\rangle$  is a fiducial normalized vector,  $\hat{X}_i$  and  $\hat{P}_i$  are representations of the generators  $X_i$  and  $P_i$  as Hermitian operators, and  $\hat{I}$  is the identity operator representing the central generator  $I$ . As a basis, the set of coherent states is overcomplete. Wavefunction  $\phi(p^i, x^i)$  for each of the state is a symmetry minimal uncertainty Gaussian centered at the state label, which correspond to expectation values rather than eigenvalues of the  $\hat{P}_i$  and  $\hat{X}_i$  observables. The operators generates translations of  $x^i$  and  $p_i$  exactly as described by the coset representation, with the nontrivial phase transformation. The  $|0, 0\rangle$  is the same as the ground state for a harmonic oscillator. The coherent states are like the classical state described in quantum mechanics.

What is particular interesting to note is that one can have the parallel construction starting from the (configuration) space coset of Eq.(4). The relevant subgroup is the one generated by  $P_i$  and  $I$ . We have a unitary representation as the span

$$e^{i\theta} |x^i\rangle = U'(x^i, \theta) |0\rangle \quad (8)$$

where

$$U'(x^i, \theta) \equiv e^{i\theta \hat{I}} e^{-ix^i \hat{P}_i}, \quad (9)$$

$|0\rangle$  is the fiducial normalized vector, and  $\hat{P}_i$  and  $\hat{I}$  the Hermitian operators generating translations in  $x^i$  and the phase rotation. Checking from the coset space action, one can see that the  $|x^i\rangle$  states are position eigenstates. We have hence a Hilbert space as the one usually presented in basic quantum mechanics textbook, which is of course unitary equivalent to the previous one from the coherent states. It looks very much like it is the quantum space we have, and the quantum configuration space for a particle is the same as its phase space.

The real quantum phase space is not the Hilbert space but its projective counterpart. Each one dimensional subspace as a ray corresponds to a distinct physical state. The projective Hilbert space is known to have the structure of an infinite dimensional symplectic manifold and Kähler manifold [10]. We can look it that through an expansion of each state in terms of the Fock state basis as

$$|\phi\rangle = \sum (q_n + ip_n) |n\rangle \quad (10)$$

for real homogeneous coordinates  $q_n$  and  $p_n$  of the projective space. The Schrödinger equation

$$i\hbar \frac{d}{dt} |\phi\rangle = \hat{H} |\phi\rangle \quad (11)$$

for the state is actually equivalent to the set of Hamilton equations

$$\begin{aligned} \frac{d}{dt} q_n &= \frac{\partial}{\partial p_n} H(p_n, q_n), \\ \frac{d}{dt} p_n &= -\frac{\partial}{\partial q_n} H(p_n, q_n), \end{aligned} \quad (12)$$

with the Hamiltonian function  $H(p_n, q_n) = \frac{2}{\hbar} \langle \phi | \phi \rangle$ . The basic fact is unfortunately much less appreciated than it should be. The projective Hilbert space also has a metric. It has no problem serving as a model of the physical space either. The countable infinite set of complex coordinates  $q_n + ip_n$  are just  $\langle n | \phi \rangle$ . We can also think of  $\phi(x^i) = \langle x^i | \phi \rangle$  as a set of complex coordinates. A functional analog of the Hamilton equations can be obtained for the real and imaginary parts of the function. The real and imaginary parts of each  $\langle x^i | \phi \rangle$  then serves as a set of real coordinates. It is obviously essentially the same for the coherent state wavefunction  $\phi(p^i, x^i) = \langle p^i, x^i | \phi \rangle$ . There are uncountable infinite number of them, hence a set with much redundancy. The picture says a quantum wavefunction is a full and definite description of the position (and momentum) of the particle. The space of all such position a free particle can possibly have, *i.e.* the full (projective Hilbert space is a model of the quantum space. Our familiar position operator  $\hat{X}_i$  describe a notion of position which is obviously the one in the classical model. The quantum particle does not have a unique position in the classical model of space for the main reason that the latter fails as a good model of the space. We can also see that it is the phase transformations generated by the central charge that makes it no longer possible to have a reduced notion of the (configuration) space from the phase space. The Lagrangian submanifold of a symplectic manifold of a classical system, as the configuration space, can be taken as the real part of the symplectic manifold with a specific complex structure. A phase transformation that mixes the real and imaginary parts is a canonical transformation which mixes the configuration and momentum variable and not a symmetry transformation of the (configuration) space itself. With the quantum symmetry, we have phase transformations within the relativity symmetry. They are hence symmetry transformation of (configuration) space. The feature is essentially dictated by the Heisenberg commutation relation.

#### 4. Classical Limit as an Approximation

We have our quantum model of the physical space. A nontrivial check that the idea is admissible is that it can explain the success of the corresponding classical picture in the proper limit. The latter is given by a symmetry contraction. More specifically, one should take the original representation which describes the quantum physics to the required limit rather than directly building the classical physics description from the contracted symmetry. We will see that the contraction of the representation does indeed give a representation of the contracted symmetry [4, 5]. Classical mechanics should be seen as such an approximation to quantum mechanics [11]. We require the quantum space model to reduce to the classical one as sort of the  $\hbar \rightarrow 0$  limit through tracing the impact of the contraction on the Hilbert space representation(s) described. We are talking about the infinite dimensional curved space as the projective Hilbert space approximated by a finite dimensional Euclidean space. It works !

The symmetry contraction is given on the Lie algebra as the  $k \rightarrow \infty$  with  $X_i^c = \frac{1}{k} X_i$  and  $P_i^c = \frac{1}{k} P_i$ . Naively, one can take  $\frac{1}{k^2}$  as  $\hbar$ . The nonzero commutator within  $H(3)$ , now in terms of  $X_i^c$ ,  $P_i^c$  and  $I$ , goes as

$$[X_i^c, P_j^c] = \frac{i}{k^2} \delta_{ij} I \rightarrow 0 . \quad (13)$$

Hence, symmetry (sub)group, as well as the full  $\tilde{G}(3)$  reduces to a trivial central extension of the classical symmetry. The  $I$  generator completely decoupled from the rest and becomes quite irrelevant. One can see that the transformation properties of the coset space representations in Eqs.(4) and (5) reduce to the classical analog as

$$H_R(3)/SO(3) \rightarrow G_s(3)/SO(3) \times U(1)$$

and

$$H_R(3)/ISO(3) \rightarrow G_s(3)/ISO(3) \times U(1) .$$

Explicitly, we have at the  $k \rightarrow \infty$  limit

$$\begin{aligned} d\theta &= \bar{\theta} , \\ dx_c^i &= \omega_j^i x_c^j + \bar{x}_c^i , \\ dp_c^i &= \omega_j^i p_c^j + \bar{p}_c^i . \end{aligned} \quad (14)$$

For the coherent state picture, we first have the  $|p^i, x^i\rangle$  states relabeled as  $|\tilde{p}_i^c, \tilde{x}_i^c\rangle$  where  $\tilde{p}_i^c = \sqrt{\hbar} p_i$  and  $\tilde{x}_i^c = \sqrt{\hbar} x_i$  ( $p_i = p^i$ ,  $x_i = x^i$ ). The new label correspond to the expectation values of  $\hat{X}_i^c$  and  $\hat{P}_i^c$ . We have then

$$\begin{aligned} \langle \tilde{p}_i^c, \tilde{x}_i^c | \hat{X}_i^c | \tilde{p}_i^c, \tilde{x}_i^c \rangle &= \frac{(\tilde{x}_i^c + \tilde{x}_i^c) - i(\tilde{p}_i^c - \tilde{p}_i^c)}{2} \langle \tilde{p}_i^c, \tilde{x}_i^c | \tilde{p}_i^c, \tilde{x}_i^c \rangle , \\ \langle \tilde{p}_i^c, \tilde{x}_i^c | \hat{P}_i^c | \tilde{p}_i^c, \tilde{x}_i^c \rangle &= \frac{(\tilde{p}_i^c + \tilde{p}_i^c) + i(\tilde{x}_i^c - \tilde{x}_i^c)}{2} \langle \tilde{p}_i^c, \tilde{x}_i^c | \tilde{p}_i^c, \tilde{x}_i^c \rangle , \end{aligned} \quad (15)$$

with

$$\langle \tilde{p}_i^c, \tilde{x}_i^c | \tilde{p}_i^c, \tilde{x}_i^c \rangle = \exp \left[ i \frac{\tilde{x}_i^c \tilde{p}_i^c - \tilde{p}_i^c \tilde{x}_i^c}{2\hbar} \right] \exp \left[ - \frac{(\tilde{x}_i^c - \tilde{x}_i^c)^2 + (\tilde{p}_i^c - \tilde{p}_i^c)^2}{4\hbar} \right] , \quad (16)$$

$$\langle \tilde{p}_i^c, \tilde{x}_i^c | \tilde{p}_i^c, \tilde{x}_i^c \rangle = 1 . \quad (17)$$

The contraction result correspond to the  $k \rightarrow \infty$  limit, hence  $\hbar \rightarrow 0$ . One can see that the last exponential factor hence the full result in Eq.(16) goes to 0. The basis states becomes mutually orthogonal. The equation above then say that the  $\hat{X}_i^c$  and  $\hat{P}_i^c$  operators become diagonal on the basis set. The conclusion there is that the original Hilbert space representation becomes a reducible representation. It reduces to a simple sum of the one dimensional spaces for the  $|\tilde{p}_i^c, \tilde{x}_i^c\rangle$  states. But such rays of the original Hilbert space each correspond to a single state. They are the only states remain in the classical limit. All the nontrivial linear superpositions are eliminated. That is to say, the original projective Hilbert space reduces to the classical six dimensional phase space.

What about the physical space or the configuration space? We can actually do better than simply saying the latter is naturally a Lagrangian subspace of the (classical) phase space. We can start from looking at the  $|x^i\rangle$  basis states we have in constructing the quantum Hilbert space as the model of the physical space and trace its contraction limit. The basis states are to be relabeled as  $|x_i^c\rangle$ , *i.e.* by eigenvalues of the  $\hat{X}_i^c$ . Now,  $\hat{P}_i^c$  and hence all operators, say, as polynomials of  $\hat{X}_i^c$  and  $\hat{P}_i^c$  commute. We have the same conclusion for the reduction of the Hilbert space now seeing as going to the simple sum of the rays of  $|x_i^c\rangle$ . The projective Hilbert space is of course the three dimensional Euclidean space.

## 5. Concluding Remarks

In physics, we are supposed to learn from experiments what constitutes a good/correct theoretical/mathematical model of any physical concept, and physical space should not be an exception. As physics advances, we see our understanding of all the intuitive concepts keeps deepening. The abstract mathematical contents to be given to such concepts changes as we replace a fundamental theory with a better one. Spacetime, or space and time, is the most basic concepts in our all discourse on physics. Einstein's relativity theories advanced our understanding of spacetime. Looking at it from the proper point of view, quantum mechanics does the same. In a way, it would be quite surprising if it does not. Our talk here offers exactly such a perspective.

A notion about spacetime being somehow emergent rather than fundamental has been getting popular lately. In many cases, it is not clear what the author(s) exactly meant about that. The way we see it: The familiar classical picture and all the related features are almost definitely not at all fundamental. They are like our classical approximate description of spacetime. It should be quite clear nowadays that the deep microscopic picture of spacetime has to be very different. We sure expect any good model of quantum spacetime model to give back the classical model as an approximation under proper limits. As such, one can certainly call the classical model a emergent one. However, to have a fundamental theory of physics built completely without some notion of spacetime in it, we think, would be quite impossible. There has to be some dynamical variables, whatever unfamiliar mathematical objects needed to described them, in the theory. For the part of such variables necessary to give a picture of the spacetime as we have in the known classical theories at the proper limit, it is only fair to call the collection of all their admissible values in the most general physical setting the spacetime model of the theory.

We would also like to point out that quantum field theory understood correctly gives another very interesting perspective about spacetime, a feature we expect to stay as a part of any model of the deep microscopic (quantum) spacetime. Quantum field theory says that spacetime is the only physical entity. The various quantum fields are not independent dynamical ‘objects’ but only dynamic degrees of freedom the spacetime has. Any state, says the state which corresponds to a single muon, involves all the degrees of freedom. With enough precision, we can see the role of any degree of freedom in any other basic properties of the state. The magnetic dipole moment of the muon has been used to reveal information about possible superparticles — postulated new quantum fields the discovery of which is one of the key target of the Large Hadron Collider experiments. We know already from quantum entanglement that the full system is more than the sum of its parts. There are no truly independent subsystems.

The above leads naturally to the question of gravitation or quantum gravity. We believe ‘*Noncommutative Geometry is to Quantum Gravity as NonEuclidean Geometry is to Gravity*’. One should build quantum gravity as a *geometrodynamics of quantum spacetime*. Our quantum space model presented here is of course very far from the final quantum spacetime model. We need to first go from the  $\tilde{G}(3)$  setting to one that incorporates Poincaré symmetry. Actually, we have a basic perspective which goes beyond that to a full stable symmetry that cannot be the contraction limit of another symmetry [4]. Our basic stable symmetry have completely noncommutative  $X_\mu$  and  $P_\mu$ . We need also to see how the above perspective of quantum field theory. We need full dynamical pictures which includes analysis of the corresponding algebras of observables [11]. We have an exciting long way to go.

Let us have a last word on basic quantum mechanics. Starting with our quantum space model, quantum mechanics can be seen from a very different light. For instance, it begs the question of why not sees as physical observables all functions of the (quantum) position and momentum, as coordinates of the quantum space/phase space. The usual notion of physical observables as essentially functions as operator  $\hat{X}_i$  and  $\hat{P}_i$  may be too limited. They are only observables which have a clear classical analog. And as such, there is no reason to restrict possible knowledge about the operator/observables on a state to their expectation values either. All the other moments of the distributions are conceptually as measurable as the expectation values. On a complementary point of view, the noncommutative algebra of quantum observables (operators) can be thought of as all continuous function  $C(X)$  of a topological space  $X$  is the general setting of noncommutative geometry [2]. Thinking about the quantum space as having noncommutative coordinates, essentially  $x_i \star$  and  $p_i \star$  (kind of  $\hat{X}_i$  and  $\hat{P}_i$ ) [11] is an alternative picture the infinite number of real number coordinates here. We are familiar with thinking about physical quantities as real number valued. That is no more than an assumption. The notion of physical quantities as having values in some other algebraic entity may be the way to go. We are familiar with the Bohr argument about all measurement being classical. That has to be the case



only if we restrict results of measurements to be real numbers. A generic measurement should only to a process to extract information from a physical system. The system being quantum says that we may want to extract from it quantum information rather than the familiar classical one. We are still learning to manipulate quantum information well. But we can do that, we can deal with true quantum measurements.

Dealing with physical quantities as beyond real number values and quantum space(time) as beyond real number geometries is like a new Copernican revolution for our generation to push on !

## Acknowledgments

The author is partially supported by research grants MOST 105-2112-M-008-017- from the MOST of Taiwan.

## References

- [1] Connes A 1994 *Noncommutative Geometry* (Academic Press)  
van Suijlekom W D 2015 *Noncommutative Geometry and Particle Physics* (Springer)
- [2] The results discussed here are based on Chew C S, Kong O C W and Payne J 2016 *Preprint NCU-HEP-k066*, *arXiv:1603.05371 [quant-ph]*
- [3] A conceptual discussion in a boarder context has been presented in the earlier article in Chinese — Kong O C W 2015 *General Relativity : A Centennial Perspective*, (Taiwan : Locus Publishing) p 185
- [4] Kong O C W 2008 *Phys. Lett. B* **665** 58  
see also Kong O C W 2008 *Preprint arXiv:0906.3581 [gr-qc]*
- [5] Cho D N and Kong O C W 2014 *Ann. Phys.* **351** 275
- [6] Gilmore R 2005 *Lie Groups, Lie Algebras, and Some of Their Applications* (Dover)  
Tung W K 1985 *Group Theory in Physics* (World Scientific)
- [7] For a detailed pedagogical presentation, see Kong O C W and Payne J *Preprint NCU-HEP-k067*
- [8] de Azcárraga J A and Izquierdo J M 1995 *Lie Groups, Lie Algebras, Cohomology and Some Applications in Physics* (Cambridge University Press).
- [9] Peremolov A M 1977 *Sov. Phys. Usp.* **20** 103  
Zhang W-M, Feng D H and Gilmore R 1990 *Rev. Mod. Phys.* **62** 867  
Klauder J R and E.C. Sudarshan E C 1968 *Fundamentals of Quantum Optics* (Benjamin)  
Klauder J R 2010 *A Modern Approach to Functional Integration* (Birkhäuser)
- [10] Bengtsson I and Życzkowski K 2006 *Geometry of Quantum States* (Cambridge University Press)
- [11] Chew C S, Kong O C W and Payne J 2017 *Preprint NCU-HEP-k070*, in preparation, gives a full presentation of the dynamical picture.